Calculus Warm Up #4-4

1. Find $\frac{dy}{dx}$ by implicit differentiation:

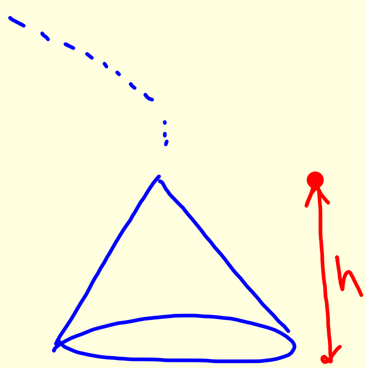
$$4x^2 - 6xy + 3 = -8y + 2xy^2 - 5$$

2. Find and simplify the acceleration equation for the movement of a particle with the given position function:

$$s(t) = (2t^2 + 3t)^3$$

HW Questions: p. 149

11. At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at the rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when it is 15 feet high?



$d = 3h$

$2r = 3h$

$r = \frac{3h}{2}$

$V = \frac{1}{3}\pi r^2 h$

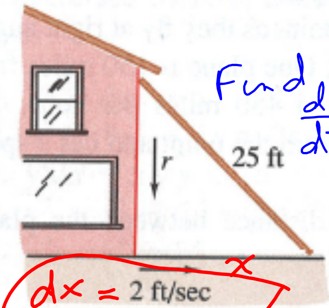
$V = \frac{1}{3}\pi\left(\frac{3}{2}h\right)^2 h$

$V = \frac{1}{3}\pi \frac{9h^3}{4}$

$V = \frac{3\pi}{4} h^3$

$\frac{dV}{dt} =$

21. A ladder 25 feet long is leaning against the wall of a house (see figure). The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top moving down the wall when the base of the ladder is (a) 7 feet, (b) 15 feet, and (c) 24 feet from the wall?



$\frac{dx}{dt} = 2$ ft/sec

$x^2 + r^2 = 25^2$

$r = \sqrt{625 - x^2}$

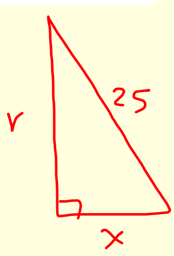
Find $\frac{dr}{dt}$

$2x\frac{dx}{dt} + 2r\frac{dr}{dt} = 0$

$\frac{dr}{dt} = \frac{-\cancel{2}x\frac{dx}{dt}}{\cancel{2}r}$

$\frac{dr}{dt} = \frac{-x}{\sqrt{625-x^2}} \cdot (2)$

23. Consider the right triangle formed by the moving ladder, the side of the house, and the ground in Exercise 21. When the base is 7 feet from the wall, find the rate at which the area of the triangle is changing.



$A = \frac{1}{2}xr$

$\frac{dA}{dt} = \frac{1}{2}x\frac{dr}{dt} + \frac{1}{2}r\frac{dx}{dt}$

$= \frac{1}{2}(7)\left(-\frac{7}{12}\right) + \frac{1}{2}(24)(2)$

$= -\frac{49}{24} + \frac{576}{24}$

≈ 21.96 ft^2/sec

when $x = 7$

$r^2 + 7^2 = 25^2$

$r = 24$

from #21: $\frac{dr}{dt} = -\frac{7}{12}$

given: $\frac{dx}{dt} = 2$

25. An air traffic controller spots two planes at the same altitude converging on a point as they fly at right angles to each other (see figure). One plane is 150 miles from the point and is moving at 450 miles per hour. The other plane is 200 miles from the point and has a speed of 600 miles per hour.

(a) At what rate is the distance between the planes decreasing? find $\frac{ds}{dt}$

(b) How much time does the traffic controller have to get one of the planes on a different flight path?

given

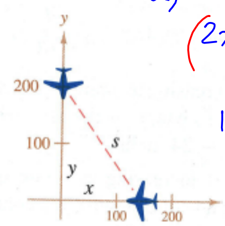
$\frac{dx}{dt} = -450$ mph

$\frac{dy}{dt} = -600$ mph

a) $x^2 + y^2 = s^2 \longrightarrow s = \sqrt{x^2+y^2}$

$s = 250$

$\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}\right) / 2$



$150(-450) + 200(-600) = 250\frac{ds}{dt}$

$\frac{ds}{dt} = -750$ mph

b) $t = \frac{d}{r} \rightarrow$ for x: $t = \frac{150}{450}$

$= \frac{1}{3}$ hr

(20 min)

for $y \rightarrow t = \frac{200}{600}$

20 min.

29. A man 6 feet tall walks at a rate of 5 feet per second away from a light that is 15 feet above the ground (see figure). When he is 10 feet from the base of the light,

(a) at what rate is the tip of his shadow moving?

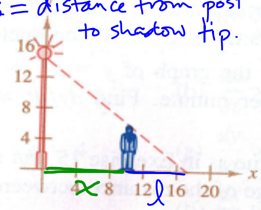
(b) at what rate is the length of his shadow changing?

l = shadow length

s = distance from post to shadow tip.

x = distance from post to man

$\frac{dx}{dt} = 5$ ft/sec



a) $\frac{15}{6} = \frac{s}{s-x}$

$5s - 5x = 2s$

$s = \frac{5}{3}x$

$\frac{ds}{dt} = \frac{5}{3}\frac{dx}{dt}$

$\frac{ds}{dt} = \frac{5}{3} \cdot 5$

$= \frac{25}{3}$ ft/sec

b) $l = s - x$

$l = \frac{5}{3}x - x$

$l = \frac{2}{3}x$

$\frac{dl}{dt} = \frac{2}{3}\frac{dx}{dt}$

$\frac{dl}{dt} = \frac{2}{3} \cdot 5$

$= \frac{10}{3}$ ft/sec

REVIEW EXERCISES for Chapter 3 p. 151

In Exercises 1–24, find the derivative of the given function.

3. $f(x) = x^{1/2} - x^{-1/2}$

5. $g(t) = \dfrac{2}{3t^2}$

7. $f(x) = \sqrt{x^3 + 1}$

9. $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$

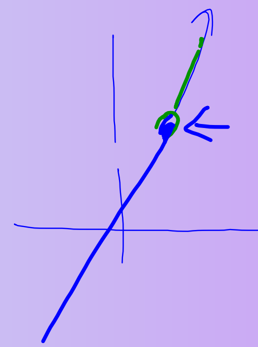
15. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

19. $f(x) = \dfrac{1}{4 - 3x^2}$

21. $g(x) = \dfrac{2x}{\sqrt{x + 1}}$

Practice (this is like the common error question on the last test)

Find the value of m so the function is continuous.



$$f(x) = \begin{cases} mx + 2, & 0 \le x \le 3 \\ x^2 + 4, & 3 < x \le 4 \end{cases}$$

$$m(3) + 2 = (3)^2 + 4$$

$$3m = 11$$

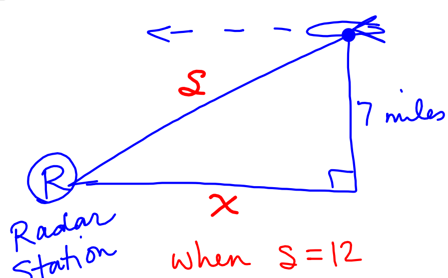
$$m = \frac{11}{3}$$

Yesterday's classwork:

An airplane is flying at an elevation of 7 miles on a flight path that will take it directly over a radar tracking station. Let *s* represent the distance (measured in miles) between the radar station and the plane. If *s* is decreasing at a rate of 420 miles per hour when *s* is 12 miles, what is the velocity of the plane?

Given: $\frac{ds}{dt} = -420$ mph

Find: $\frac{dx}{dt}$



$$x^2 + 7^2 = s^2$$

$$\frac{2x\frac{dx}{dt} + 0}{2x} = \frac{2s\frac{ds}{dt}}{2x}$$

$$\frac{dx}{dt} = \frac{s}{x} \cdot \frac{ds}{dt}$$

when $s = 12$

$$x = \sqrt{144 - 49}$$

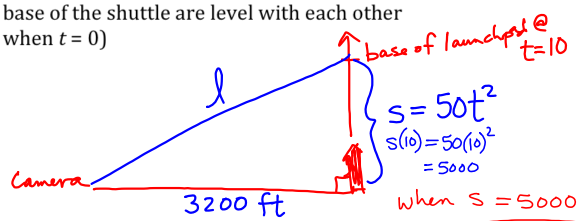
$$x = \sqrt{95}$$

$$\frac{dx}{dt} = \frac{12}{\sqrt{95}}(-420)$$

$$\frac{dx}{dt} \approx -517.09 \text{ mph}$$

A television camera at ground level is filming the lift-off of a space shuttle that is rising vertically according to the position equation $s = 50t^2$, where s is measured in feet and t is measured in seconds. The camera is 3200 feet from the launch pad. Find the rate of change in the distance between the camera and the base of the shuttle 10 seconds after lift-off. (Assume that the camera and the base of the shuttle are level with each other when $t = 0$)

Find: $\frac{d\ell}{dt}$



$s = 50t^2$

$s(10) = 50(10)^2$

$= 5000$

when $s = 5000$

$\ell = \sqrt{3200^2 + 5000^2}$

$= 200\sqrt{881}$

(or store your decimal ☺)

$(3200)^2 + s^2 = \ell^2$

$0 + 2s\frac{ds}{dt} = 2\ell\frac{d\ell}{dt}$

$\frac{2s\frac{ds}{dt}}{2\ell} = \frac{2\ell\frac{d\ell}{dt}}{2\ell}$

$\frac{d\ell}{dt} = \frac{s}{\ell}\frac{ds}{dt}$

plug in $t = 10$

$\frac{d\ell}{dt} = \frac{5000}{200\sqrt{881}}(100t)$

$\frac{d\ell}{dt} = \frac{25000}{\sqrt{881}} \approx 842.3\ \frac{ft}{sec}$

A trough is 12 feet long and 3 feet across the top. Its ends are isosceles triangles with an altitude of 3 feet. If water is being pumped into the trough at 2 cubic feet per minute, how fast is the water level rising when it is 1 foot deep?

for 

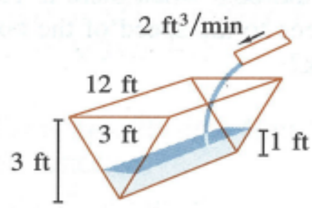
$b = h$

Triangular Prism

$V = (\text{Area}_\Delta)(h)$

$V = \frac{1}{2}bh \cdot 12ft$

$V = 6h^2$



given:

$\frac{dV}{dt} = 2\ ft^3/min$

$\frac{dV}{dt} = 12h\frac{dh}{dt}$

$2 = 12(1)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{6}\ ft/min$

Test Monday:

- Differentiation with
  - Power Rule
  - Product Rule
  - Quotient Rule
  - Chain Rule

Implicit Differentiation

Particle Motion: Position, velocity, acceleration

Tangent Lines

Related Rates

HW: Chapter review

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and Ch. 3 Review WS